

Field precision and performance

John Apostolakis

representing the effort of
the Geometry & Transportation WG
of Geant4

Version 0.41

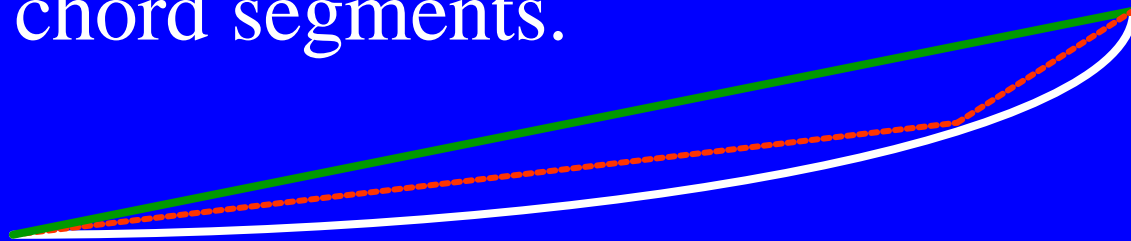
12th November 2002

Magnetic field: overview

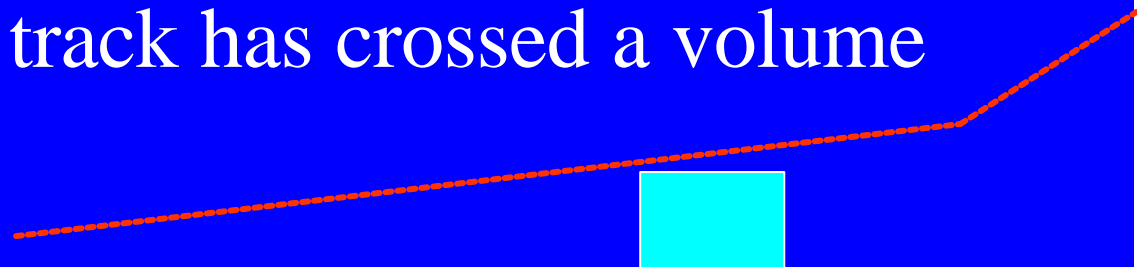
- In order to **propagate** a particle inside a field (e.g. magnetic, electric or both), we **solve** the equation of motion of the particle in the field.
- We use a Runge-Kutta method for the integration of the ordinary differential equations of motion.
 - Several Runge-Kutta ‘steppers’ are available.
- In specific cases other solvers can also be used:
 - In a uniform field, using the analytical solution.
 - In a nearly uniform field (BgsTransportation/future)
 - In a smooth but varying field, with new RK+helix.

Magnetic field: overview (cont)

- Using the method to calculate the track's motion in a field, Geant4 breaks up this curved path into linear chord segments.

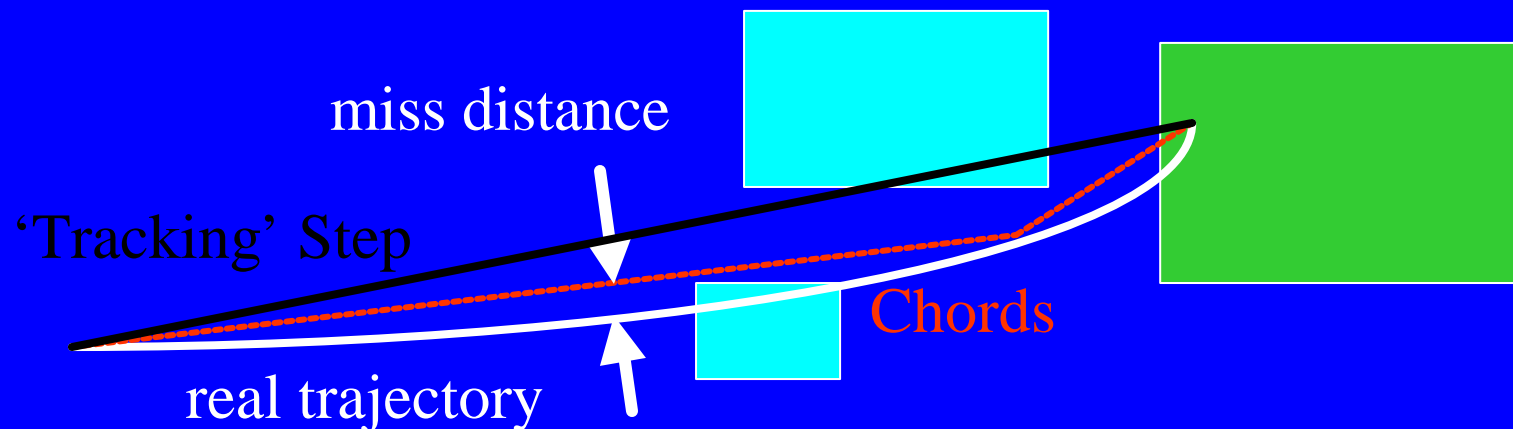


- We determine the chord segments so that they closely approximate the curved path.
- We use the chords to interrogate the Navigator, to see whether the track has crossed a volume boundary.



Stepping and accuracy

- You can set the accuracy of the volume intersection,
 - by setting a parameter called the “miss distance”
 - it is a measure of the error in whether the approximate track intersects a volume.
 - Default “miss distance” is 3 mm.
- One physics/tracking step can create several chords.
 - In some cases, one step consists of several helix turns.



Global and local fields

- One field manager is associated with the ‘world’
 - Set in G4TransportationManager
- Other volumes can override this
 - By associating a field manager with any logical volume
 - By default this is propagated to all its daughter volumes

```
G4FieldManager* localFieldMgr=  
    new G4FieldManager(magField);  
logVolume->setFieldManager(localFieldMgr,  
    true);
```

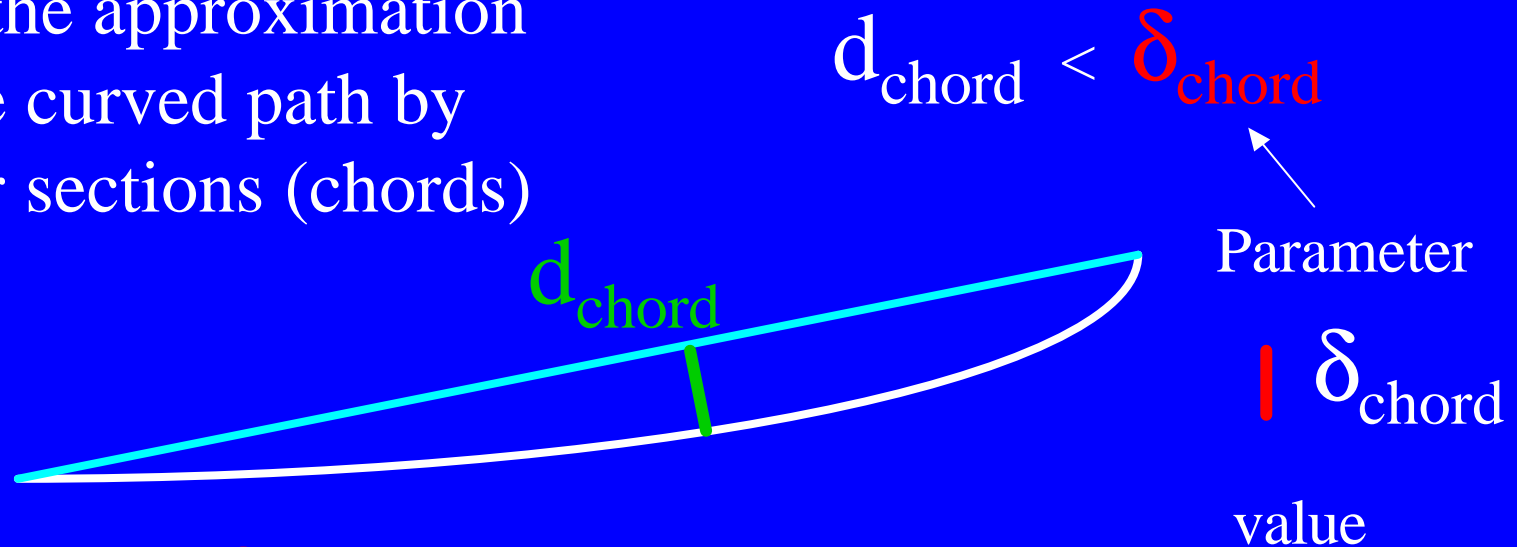
where ‘true’ makes it push the field to all the volumes it contains.

Precision parameters

- Errors come from
 - Break-up of **curved** trajectory into **linear** chords
 - Numerical **integration** of equation of motion
 - or potential approximation of the path,
 - **Intersection** of path with volume boundary.
- Precision parameters enable the user to limit these errors and control performance.
 - The following slides attempt to explain these parameters and their effects.

Volume miss error

Due to the approximation
of the curved path by
linear sections (chords)



- Parameter δ_{chord}
- Effect of this parameter as $\delta_{\text{chord}} \rightarrow 0$

$$S_{1\text{step}}^{\text{propagator}} \cong (8 \delta_{\text{chord}} R_{\text{curv}})^{1/2}$$

so long as $s^{\text{propagator}} < s^{\text{phys}}$ and $s^{\text{propagator}} > d_{\text{min}}^{\text{integr}}$

Integration error

Due to error in the numerical integration (of equations of motion)

Parameter(s): $\epsilon_{\text{integration}}$

$$\max(\|\vec{\Delta r}\| / s_{\text{step}}, \|\vec{\Delta p}\| / \|\vec{p}\|) < \epsilon_{\text{integration}}$$

$s_{1\text{step}}$

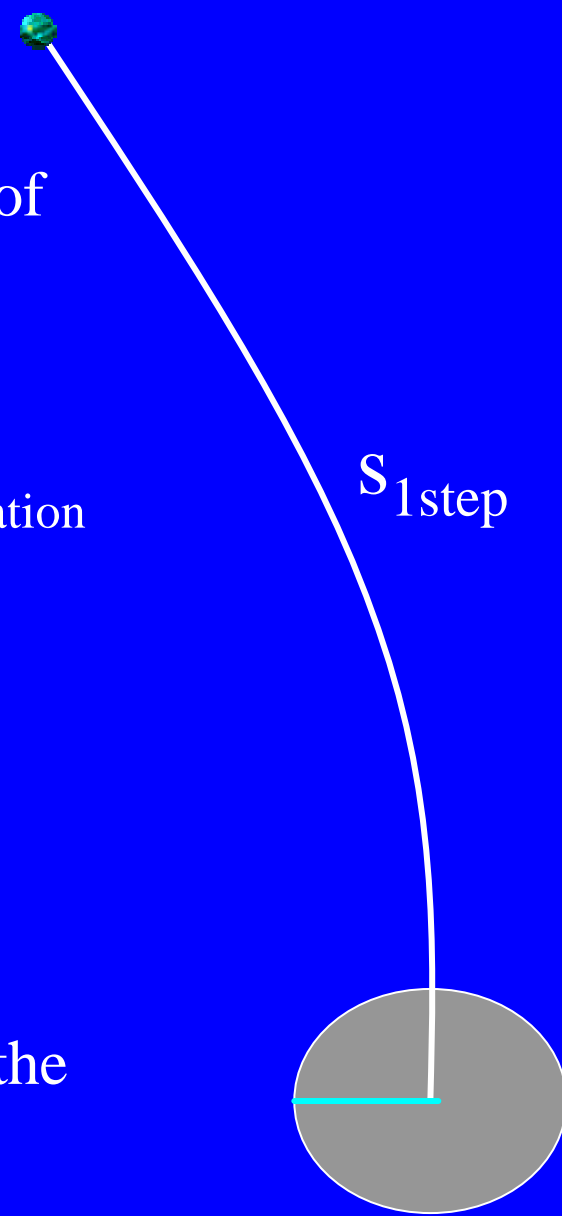
- It limits the size of the integration step.
- For Classical RK4 Stepper

$$s_{1\text{step}}^{\text{integration}} \sim (\epsilon_{\text{integration}})^{1/3}$$

for small enough $\epsilon_{\text{integration}}$

- The integration error should be influenced by the precision of the knowledge of the field (measurement or modeling).

$$N_{\text{steps}} \sim (\epsilon_{\text{integration}})^{-1/3}$$



Integration errors (cont.)

- In practice

$\epsilon_{\text{integration}}$ is currently represented by 3 parameters

- | | |
|---|----------------------------------|
| – epsilonMin, a minimum value (used for big steps) | Defaults
0.5*10 ⁻⁷ |
| – epsilonMax, a maximum value (used for small steps) | 0.05 |
| – DeltaOneStep, a distance error (for intermediate steps) | 0.25 mm |

$$\epsilon_{\text{integration}} = \delta_{\text{one step}} / S_{\text{physics}}$$

- Determining a reasonable value

- I suggest it should be the minimum of the ratio (accuracy/distance) between sensitive components, ..

- Another parameter

- | | |
|---|--------------------|
| – d_{min} is the minimum step of integration | Default
0.01 mm |
| • (newly enforced in Geant4 4.0) | |

Intersection error

- In intersecting approximate path with volume boundary
 - In trial step AB, intersection is found with a volume at C
 - Step is broken up, choosing D, so

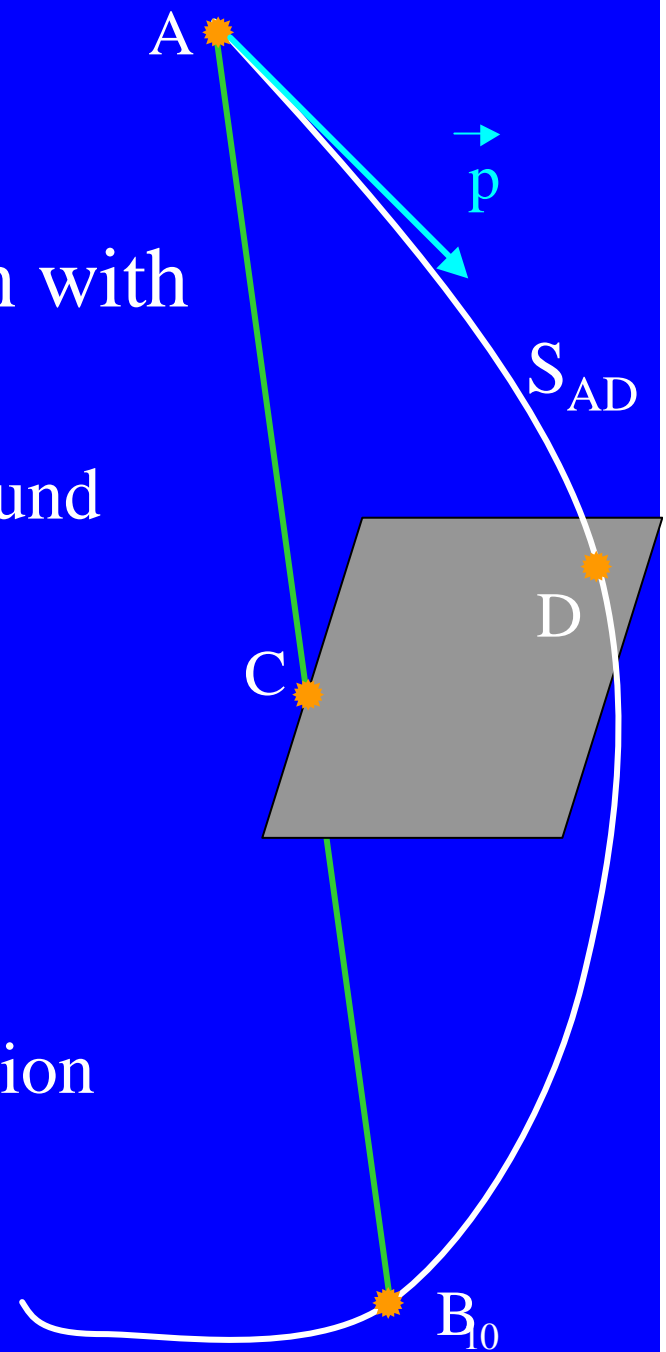
$$S_{AD} = S_{AB} * |AC| / |AB|$$

- If $|CD| < \delta_{\text{intersection}}$
 - Then C is accepted as intersection point.

– So δ_{int} is a position error/bias

13 November 2002

Geant4 Fields, J. Apostolakis, Geant4
Users Workshop, SLAC

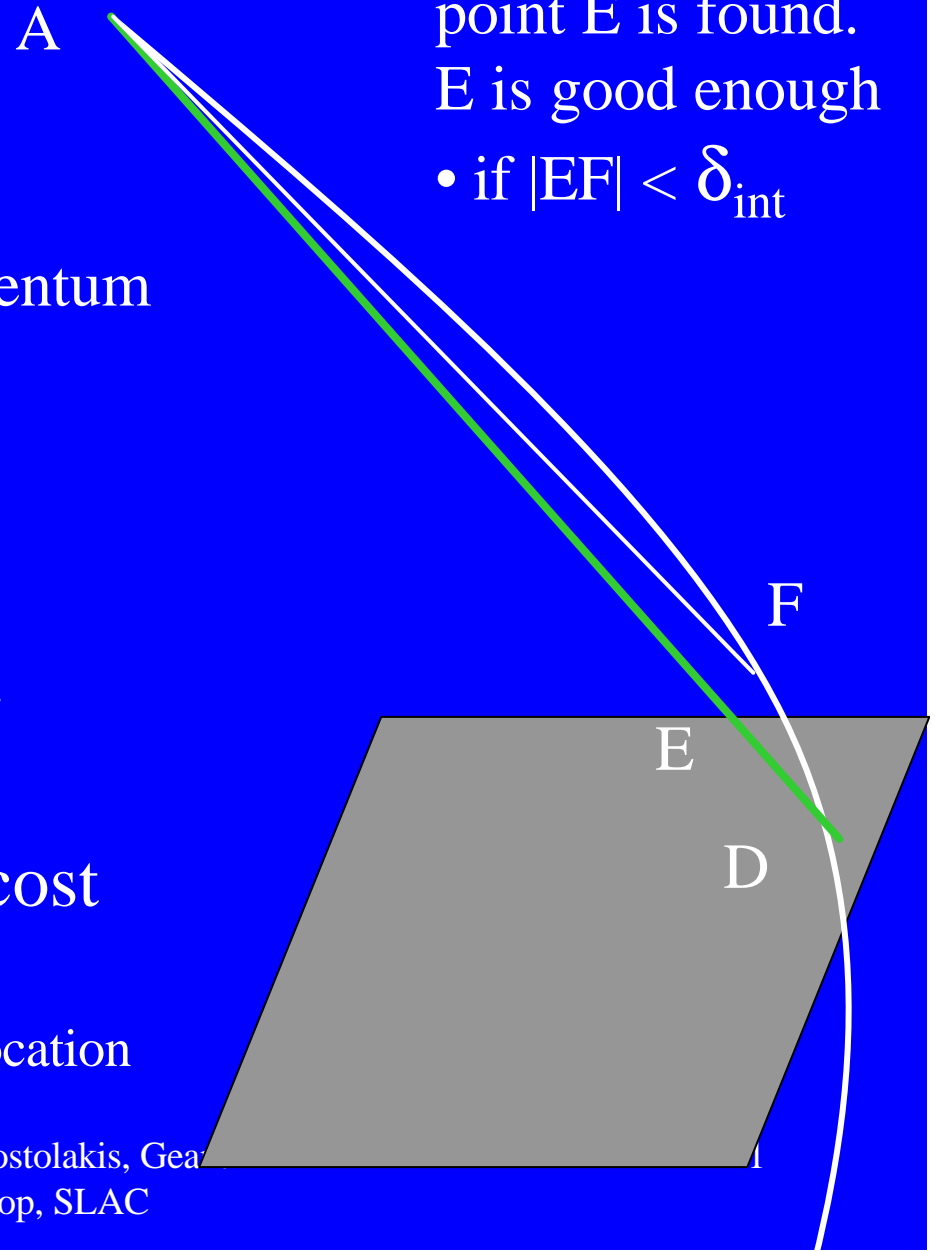


Intersection error (cont)

- So δ_{int} must be small
 - compared to tracker hit error
 - Its effect on reconstructed momentum estimates should be calculated
 - And limited to be acceptable
- Cost of small δ_{int} is less
 - than making δ_{chord} small
 - Is proportional to the number of boundary crossings – not steps.
- Quicker convergence / lower cost
 - Possible with optimization
 - adding std algorithm, as in BgsLocation

If C is rejected,
a new intersection
point E is found.
E is good enough

- if $|EF| < \delta_{\text{int}}$



The 'driving force'

- Distinguish cases according to the factor driving the tracking step length
 - 'physics', eg in dense materials
 - fine-grain geometry
- Distinguish the factor driving the propagator step length (if different)
 - Need for accuracy in 'seeing' volume
 - Integration inaccuracy
 - Strongly varying field

Potential
Influence

G4 Safety
improvement

Other Steppers,
tuning d_{\min}

What if time does not change much?

- If adjusting these parameters (together) by a significant factor (10 to 100) does not produce results,
 - Then it is likely that the field propagation is not the dominant (most CPU intensive) part of your program.
 - Look into alternative measures
 - modifying the physics ‘cuts’ – ie production thresholds
 - To create fewer secondaries, and so track fewer particles
 - determining the number of steps of neutral vs charged particles,
 - To find whether neutrons, gammas ‘dominate’
 - profiling your application
 - You can compile using G4PROFILE=yes, run your program and

Where to find the parameters

Parameter	Name	Class	Default
δ_{miss}	DeltaChord	ChordFinder	3.0 mm
d_{min}	stepMinimum	ChordFinder	0.01 mm
$\delta_{\text{intersection}}$	DeltaIntersection	FieldManager	0.10 mm
$\delta_{\text{one step}}$	DeltaOneStep	FieldManager	0.25 mm
	epsilonMin	PropagatorInField	$5 \cdot 10^{-7}$
	epsilonMax	PropagatorInField	0.05

Contributors to Field sub-category

- John Apostolakis
- Simone Giani
- Vladimir Grichine
- Wolfgang Wander

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- and expected contribution