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Development of the Fritiof Model in Geant4

Vladimir UZHINSKY [1,2*]
On behalf of the Geant4 Hadronics Working Group

[1] *CERN, CH-1211, Geneva 23, Switzerland*
[2] *LIT, JINR, 141980 Dubna, Russia*

New features are added to the Fritiof model implemented in Geant4: a simulation of elastic hadron-nucleon scatterings; a simulation of binary reactions like NN→NΔ in hadron-nucleon interactions; a separate simulation of single diffractive and non-diffractive events; the reggeon theory inspired model of nuclear destruction (RTIM) and the algorithm of "Fermi motion" simulation in nuclear reactions. These allow to describe after a parameter's tuning the energy dependencies of pp-interaction reaction cross sections ($pp\rightarrow pp\pi^0$, $pp\rightarrow np\pi^+$, $pp\rightarrow pp\pi^+\pi^-$, $pp\rightarrow np2\pi^+\pi^-$, $pp\rightarrow pp\pi^+\pi^-\pi^0$) and the HARP-CDP data on pion and proton productions in the interactions of projectile protons with momenta 3, 5, 8, 12 and 15 GeV/c with Be, Cu, Ta and Pb targets. Thus a smooth transition between the low energy hadronic models of Geant4 (Bert, Bic) and the high energy QGS model of Geant4 can be provided.

KEYWORDS: Geant4, hadron-nucleon interactions, hadron-nucleus interactions, high energies, theoretical models, Monte Carlo simulation, Fritiof model

I. Introduction

The Fritiof model[1,2)] assumes that all hadron-hadron interactions are binary reactions, $h_1+h_2\rightarrow h_1'+h_2'$, where h_1' and h_2' are excited states of the hadrons with continuous mass spectra (see Fig. 1). If one of the hadrons is in the ground state ($h_1+h_2\rightarrow h_1+h_2'$) the reaction is called "single diffraction dissociation", in other case – "non-diffractive interactions".

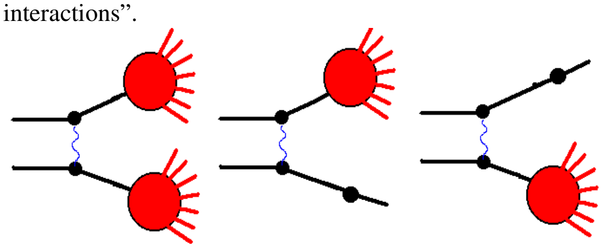

Fig. 1 Non-diffractive and diffractive interactions considered in the Fritiof model.

The excited hadrons are considered as QCD-strings, and the corresponding LUND-string fragmentation model is applied for a simulation of their decays.

The key ingredient of the Fritiof model is a sampling of the string masses. In general, a set of final state of interactions can be represented by the Fig. 2, where samples of possible string masses are shown. There is a point corresponding to an elastic scattering, a group of points which represents final states of binary hadron-hadron interactions, lines corresponding to the diffractive interactions, and various intermediate regions. The region populated with the red points is responsible for the non-diffractive interactions. In principle, the mass sampling threshold can be below hadron masses. In the model it equals to the ground state masses. The string masses are sampled in the triangular region restricted by the diagonal line corresponding to the kinematical limit $M_1+M_2=E_{cms}$, and the threshold lines. If a point is below string mass threshold, it is shifted to a nearest diffraction line.

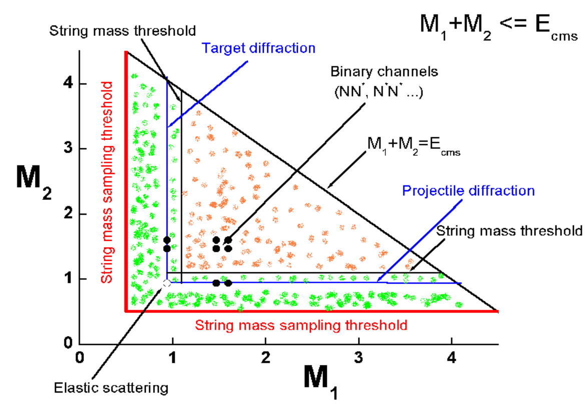

Fig. 2 Diagram of final states of hadron-hadron interactions.

In the code of the Fritiof model implemented in Geant4 we take into account binary reaction final states different from the elastic scattering. Weights of the hN elastic scattering and the hN diffractive interactions have been introduced which give probabilities of the corresponding reactions. In the case of diffraction dissociation we sample the points of the diagrams only on the diffraction lines.

The Fritiof model assumes that in the course of a hadron-nucleus interaction the string originated from the projectile can interact with various intra-nuclear nucleons and becomes into highly excited states. The probability of the multiple interactions is calculated in the simplest approximation. A cascading of secondary particles is neglected as a

*E-mail:vuzhinsk@cern.ch

rule. Due to these, the original Fritiof model fails to describe a nuclear destruction and slow particle spectra. In order to overcome the difficulties we enlarge the model by the reggeon theory inspired model of nuclear desctruction[3,4)]. Momenta of the nucleons ejected from a nucleus are sampled according to a "Fermi motion" algorithm presented in Ref. 5.

II. Simulation of nucleon-nucleon interactions

1. Simulation of the binary channels

The probability of the elastic hadron-nucleon scattering is equal to σ_{el}/σ_{tot}, where σ_{el} and σ_{tot} are elastic and total hadron-nucleon cross sections. We use PDG-92[6)] parameterizations for the cross sections. The transverse momentum transferred in the reaction is sampled according to the distribution – $\exp(-B\,P_T^2)$, $B=\sigma_{tot}^2/(16\pi\,\sigma_{el}\,0.3894)$ $(\mathrm{GeV/c})^{-2}$.

There is no quantum number exchange between colliding hadrons in the model. We introduce unlike quark exchange between hadrons. Additional to this, we have to assume that one of the final hadrons will be in the ground state. The probability of the exchange in the target rest frame is given by

$$W_{qe}=A\bullet\exp(-a\,Y_{lab}),\ A=1.84,\ a=0.7.$$

Here Y_{lab} is a rapidity of the projectile. Δ-isobars are created due to these in the pp-interactions. We simulate elastic scattering of the isobars after the quark exchange in order to generate a transferred momentum.

A Δ-isobar can be in the ground state after the exchange with the probability $W_{gs}=C\bullet\exp(-c\,Y_{lab})$, C=1.97, c=0.5. In other case we assume that it will be in an excited state, and simulate in the case a single diffraction dissociation of the isobar.

2. Simulation of the single diffraction

The probability of the single diffraction (W_{sd}) is equal to $(1-W_{qe})\bullet D\exp(-d\,Y_{lab})$, D=1.61, d=0.35. A squared mass of the string created in the reaction is equal to

$$(m')^2=P\bullet[E_{CMS} - (m_0^2 + P_T^2)/(E_{CMS} - P)] - P_T^2,$$

where P is light-cone-momentum sampled according to the distribution dP/P. $P=E-P_z$ for projectile originated hadron, and $P=E+P_z$ for target originated hadron in CMS. m_0 is a mass of hadron left in the ground state. P_T^2 is a square of the transferred transverse momentum, $\langle P_T^2\rangle=0.15$ $(\mathrm{GeV/c})^2$.

3. Simulation of the non-diffractive interactions

The probability of the non-diffractive events $W_{nd}=1-W_{qe}-W_{sd}$. P^- momentum of the projectile originated hadron and P^+ momentum of the target originated hadron are sampled independently according to the distribution dP/P.

4. Description of experimental data

Most of the parameters given above were determined at analysis of experimental data presented in Ref. 7. In Fig. 3 we compare our results with the cross section of the reaction $pp\to n p\pi^+$ where the quark exchange is a dominant process. As seen, we describe it quite well.

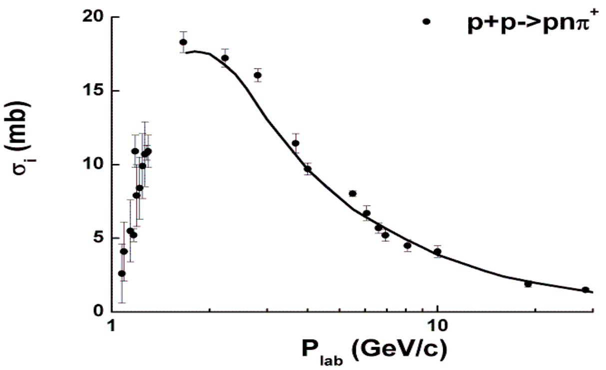

Fig. 3 Cross section of the reaction $pp\to np\pi^+$. Points are the experimental data of Ref. 7, line presents our calculation.

The process $pp\to pp\pi^0$, see Fig. 4 as the process $pp\to pp\pi^+\pi^-$, is determined by the single diffraction dissociation. It is suppressed at low energies by the quark exchange. It decreased also at energy growth as W_{sd}.

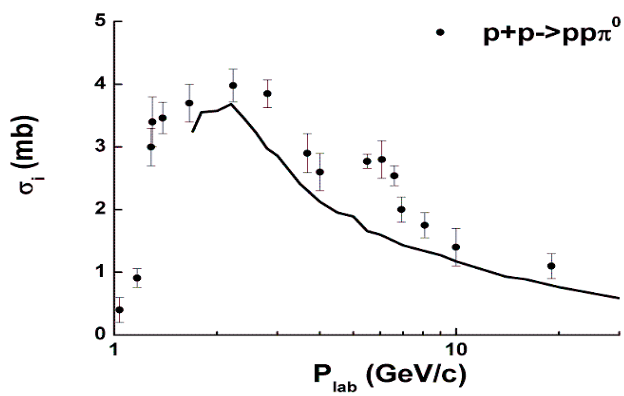

Fig. 4 Cross section of the reaction $pp\to pp\pi^0$. Points are the experimental data of Ref. 7, line presents our calculation.

The model does not describe sufficiently well the reactions $pp\to np2\pi^+\pi^-$, $pp\to pp\pi^+\pi^-\pi^0$, see Fig. 5. At present time we have no idea how to improve the description.

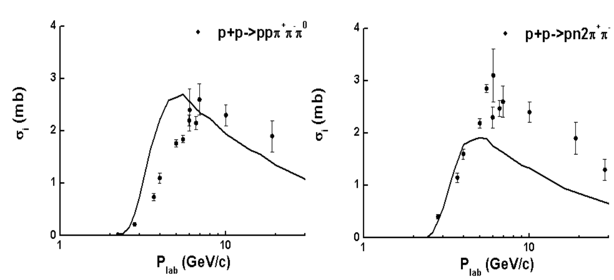

Fig. 5 Cross sections of the reactions $pp\to np2\pi^+\pi^-$, $pp\to pp\pi^+\pi^-\pi^0$. Points are the experimental data of Ref. 7, lines are our calculation.

III. Simulation of proton-nucleus interactions

1. Description of pion spectra in p+Be interactions

It is assumed that a cascading of secondary particles does not play much role in the interactions with light nucleus. Thus we can use the corresponding experimental data to verify the Fritiof model for pp-interactions. The data were obtained by the HARP-CDP group[8)], and some of them are presented in Fig. 6, 7.

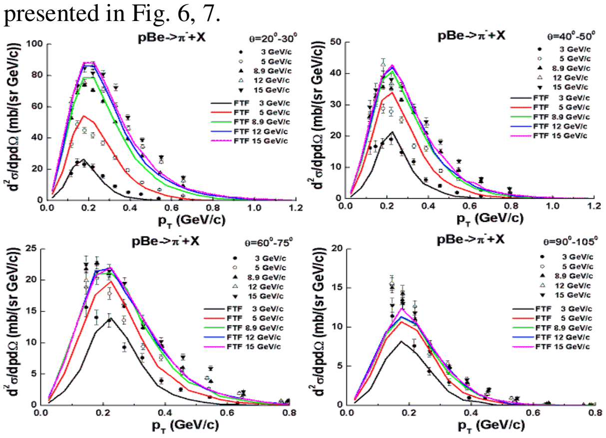

Fig. 6 Spectra of π^--mesons in p+Be interactions. Points are the experimental data of Ref. 8, lines are our calculations. The black, red, green, blue and magenta lines correspond to the calculations at P_{lab}=3, 5, 8.9, 12 and 15 GeV/c, correspondently.

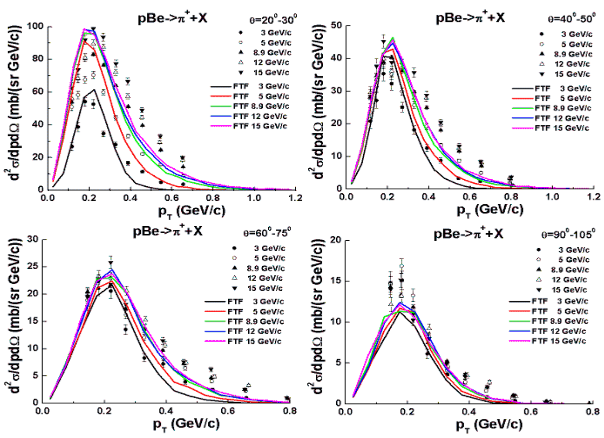

Fig. 7 Spectra of π^+-mesons in p+Be interactions. Points are the experimental data of Ref. 8, lines are our calculations.

In the Fig. 6, we present a description of the π^--meson spectra in our model. As seen, we describe them quite well. The π^+-meson spectra shown in the Fig.7 are described as well except the data at P_{lab}=5 GeV/c and θ=20 – 30 degree. To reach the agreement, we tune only one parameter of the string fragmentation, W_{LUND}, which is responsible for an end of the fragmentation. W_{LUND}=230 MeV at our definition of a minimal string mass (see code for more details).

Summing up, we have shown that the modified Fritiof model reproduces properties of pN-interactions in the energy range 2 – 15 GeV.

A direct application of the model to heavy target nuclei gave unsatisfactory results – multiplicities of produced particles are too high especially at high energies. Thus we introduced a correction of the inelastic interaction number in a nuclear medium

2. Correction of interaction number

It is obvious that the Fritiof model as other string model can not work for arbitrary nuclei at an arbitrary energy. Let us image an interaction of a particle with a neutron star. The interaction will be only in a thin skin of the matter, and will not have a longitudinal size equal to a length of possible path of the particle in the star as it is assumed in the current versions of the models. Let us call the thickness of the skin as a "longitudinal interaction length".

It is reasonable to assume that at low energies the length is lower than the average distance between nucleons in a nucleus (~ 2 fm). Thus, the usual cascading takes place. At higher energies the length will be larger, and two or more nucleons can be in the interaction region. Only these nucleons will participate in the interactions describing by the string models. Let us introduce a maximum number of the possible participating nucleons – N_{max}. Because the number grows with energy rising, it seems natural to assume that N_{max}.≈P_{lab}/P_0, where P_0 is a new parameter.

An algorithm implementing of the idea look like that: at the beginning, a projectile has a power, Pw, to interact with N_{max} nucleons (Pw=N_{max}), thus a probability of an interaction with the first nucleon, Pw/N_{max}, is equal to 1. The power decreases after the interaction on 1. The probability of an interaction with the second nucleon is equal to Pw /N_{max}, where Pw=N_{max}-1. If the second interaction is happened, the power is decreased one more. In other case, it is left on the same level. This is applied for each possible interaction.

A tuning of only one parameter, P_0, gave the value – 6 GeV/c. It allows one to describe the HARP-CDP data, see Fig. 8. As seen, the description is quite well at small agles, lower than 40°. The description at large angles can be improved if one use the Fritiof model coupled with the binary cascade model of Geant4. The binary model gives additional

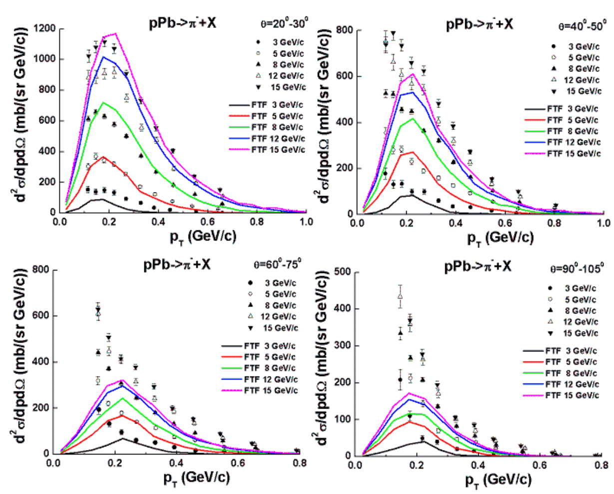

Fig. 8 Spectra of π^--mesons in p+Pb interactions. Points are the experimental data of Ref. 9, lines are our calculations.

cascading of low energy secondaries in nuclei.

3. Tuning of nuclear destruction parameters

As known, the Glauber-like approximation used in the Fritiof model and in the other string model does not provide enough amount of intra-nuclear collisions for a correct description of a nuclear destruction. To overcome the problem it is needed to take into account so-called enhanced diagrams of the reggeon theory. An attempt to do this was undertaken in Ref. 3, 4. According to the Ref. 3, 4, nucleons participating in the interaction predicted by the approximation are considered as primary ones. They can involve other spectator nucleons in the process. The process takes place only in the impact parameter space, and it is illustrated by Fig. 9.

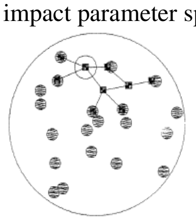

Fig. 9 Reggeon "cascade" in hA-scattering in the impact parameter plane. The position of the projectile hadron is marked by an open circle, the positions of nuclear nucleons by closed circles, reggeon exchanges by straight lines and the square points are the coordinates of the reggeon interaction vertices.

The cascade is governed by a probability to involve a new nucleon in the interaction: the spectator nucleon having an impact distance b from a 'wounded' nucleon can be involved in the process with a probability $W=C_{nd}\bullet\exp(-b^2/R^2_{nd})$, where $R_{nd}\approx 1.5$ (fm^2) is the mean interaction radius and C_{nd} is a strength factor.

We have tuned C_{nd} using the HARP-CDP data on proton production in the p+Cu interactions[10]. According to our estimations,

$$C_{nd}=0.47\bullet e^{2(y-2.5)}/(1+e^{2(y-2.5)}),$$

where y is a projectile rapidity. The value, 2.5, standing in the exponents corresponds to $P_{lab}\approx 6$ GeV/c. Let us mark that the last value coincides with the parameter P_0 introduced above.

4. Parameters of "Fermi motion" simulation algorithm

To ascribe momenta of the nucleons involved in the reggeon cascading we use an algorithm proposed in Ref. 5. It has two parameters – a widths of light-cone-momentum distribution of nucleons, dx, and an average P_T^2 of the nucleons. We have tuned the parameters using the HARP-CDP data on pCu interactions.

$$dx=0.4/A, \qquad P_T^2=0.035+0.1\bullet e^{4(y-2.5)}/(1+e^{4(y-2.5)}),$$

where A is mass number of a target nucleus. The value, 2.5, standing in the exponents corresponds to $P_{lab}\approx 6$ GeV/c.

5. Description of proton spectra in the interactions with heavy nuclei

Having implemented the reggeon cascading and the nucleon momentum sampling algorithm, we reach a good description of the proton spectra presented in Fig. 10.

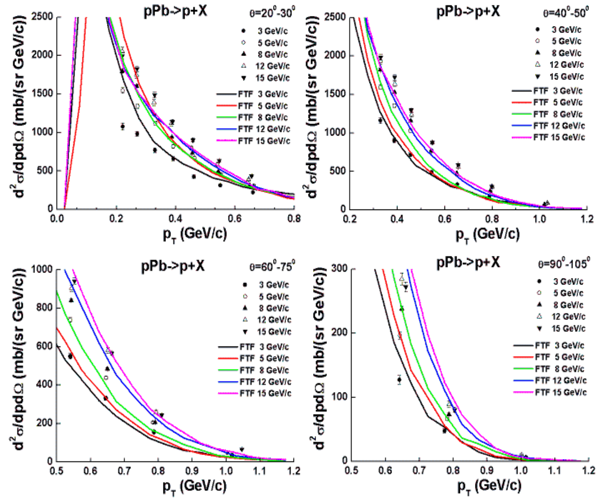

Fig. 10 Spectra of protons in p+Pb interactions. Points are the experimental data of Ref. 9, lines are our calculations.

A description of the proton spectra in p+Be interactions using the same model is not sufficiently well. We believe that it is connected with too large "Fermi momentum". It is obvious that Fermi momenta for light and heavy nuclei are different. Thus, a description of proton interactions with light nuclei requires a special consideration.

III. Conclusion

The Fritiof model of Geant4 has been improved. New ideas have been proposed and implemented. All of these allows one a good description of hadron-nucleus interactions at $P_{lab}>3$ GeV/c.

We believe that there is a change in the hadron production mechanism at $P_{lab}\approx 6$ GeV/c. First of all, the correction of the intra-nuclear interaction number requires this value. The nuclear destruction parameter C_{nd} changes essential at the value. Needed P_T^2 of the Fermi motion requires the corresponding parameter. A nature of the change (transition?) is unknown! Many Monte Carlo models (UrQMD, MCMPX, LAQGSM, DPMJET and so on) assume such transition, but they never consider it in details, and its nature. We hope the Experiment will say its last word.

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